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Peristaltic flow of a Williamson fluid in an asymmetric channel through porous medium

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Abstract:Peristaltic transport of a Williamson fluid in an asymmetric channel through porous medium is studied under long wavelength and low Reynolds number assumptions. The nonlinear governing equations of the peristaltic flow are solved using perturbation method. The solution for the stream function is obtained by neglecting inertia and curvature effects. The velocity distribution, the volume flow rate and the pressure rise are also determined.

Key words: Williamson fluid, Reynolds number, Peristalsis, Velocity and Pressure rise.

I. Introduction

Peristalsis is a well-known mechanism for pumping biological and industrial fluids. Even though it is observed in living systems for many centuries; the mathematical modeling of peristaltic transport has begun with the important works by Fung and Yih [1] using laboratory frame of reference and Shapiro et al.[2] using wave frame of reference. Many of the contributors to the area of peristaltic pumping have either followed Shapiro or Fung. Most of the studies on peristaltic flow deal with Newtonian fluids. The complex rheology of biological fluids has motivated investigations involving different non-Newtonian fluids. Peristaltic flow of nonNewtonian fluids in a tube was first studied by Raju & Devanathan [3].

Peristalsis is a mechanism adopted by many physiological systems and mechanical peristaltic pumps. Most of the physiological systems may be approximated as symmetric ducts. In view of this, Peeyush Chandra [4], Sarojamma et al [5], Ramachandra Rao and Usha [6], Misra & Pandey [7], Vajravelu et al. [8-11], Subba Reddy et al. [12, 13] and Srinivas et al.[14,15] made detailed studies on peristaltic pumping through tubes and channels. Brassuer and Anupampal (vide Chengel & Cimbala, [16]), made experiments on the mechanical functioning of the stomach using MRI (Magnetic Resonance Image). They observed that the stomach is a mixer, a grinder, a storage chamber, and a sophisticated peristaltic pump that controls the release of liquid and solid gastric content into the small intestines where nutrient uptake occurs. The MRI image of the stomach has revealed its asymmetry nature. Another physiological system namely uterus is also modeled as an asymmetric channel by Eytan and Elad [17]. These facts will explain the necessity of considering the physiological system to be asymmetric ducts also. Motivated by these facts, it will be interesting to study the peristaltic transport of Williamson fluid through an asymmetric channel filled with porous material.

In this paper peristaltic pumping of Williamson fluid through a porous medium in an asymmetric channel with flexible walls is investigated. Using the wave frame of analysis, boundary value problem is solved and the results are discussed through graphs.

II. Mathematical Formulation

Let us consider the peristaltic transport of an incompressible Williamson fluid in a two dimensional channel of width $\bar{d}_1 + \bar{d}_2$. The flow is generated by sinusoidal wave trains propagating with constant speed c along the channel walls. The geometry of the wall surfaces are defined as

$$Y = H_1 = \bar{d}_1 + \bar{a}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right] \qquad (upper \quad wall),$$

$$Y = H_2 = -\bar{d}_2 - \bar{b}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right] \quad (lower \quad wall),$$

(1)

where $\bar{a}_1$ and $\bar{b}_1$ are the amplitudes of the waves, λ is the wave length, $\bar{d}_1 + \bar{d}_2$ is the width of the channel, c is the velocity of propagation, $\bar{t}$ is the time and $\bar{X}$ is the direction of wave propagation. The phase difference ϕ varies in the range $0 \le \phi \le \pi$ in which $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase, further $\bar{a}_1, \bar{b}_1, \bar{d}_1, \bar{d}_2$ *and* ϕ satisfies the condition $\bar{a}^2{}_1 + \bar{b}^2{}_1 + 2\bar{a}_1\bar{b}_1 \cos\phi \le (\bar{d}_1 + \bar{d}_2)^2$.

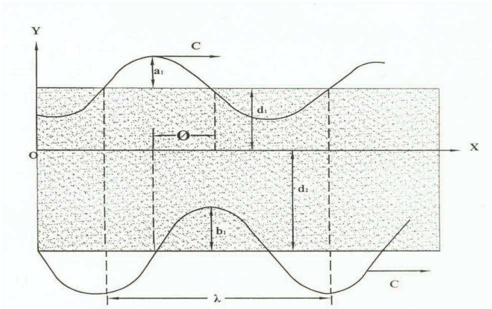

Figure 1.Physical Model

Introducing a wave frame $(\bar{x}, \bar{y})$ moving with velocity c away from the fixed frame $(\bar{X}, \bar{Y})$ by the transformation,

$$\bar{x} = \bar{X} - c\bar{t} \quad , \bar{y} = \bar{Y} \ , \bar{u} = \bar{U} - c \ , \bar{v} = \bar{V} \quad and \quad \bar{P}(x) = \bar{P}(X, t). \tag{2}$$

and defining

$$x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{\bar{d}_1}, u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c}, t = \frac{c}{\lambda}\bar{t}, \ h_1 = \frac{\bar{h}_1}{\bar{d}_1}, \ h_2 = \frac{\bar{h}_2}{\bar{d}_2}, \ \tau_{xx} = \frac{\lambda}{\mu_0 c}\bar{\tau}_{xx}, \ \tau_{xy} = \frac{\bar{d}}{\mu_0 c}\bar{\tau}_{xy}$$

$$\tau_{yy} = \frac{\bar{d}}{\mu_0 c}\bar{\tau}_{yy}, \ \delta = \frac{\bar{d}}{\lambda}, \ \mathrm{Re} = \frac{\rho c \bar{d}}{\mu_0}, We = \frac{\Gamma c}{\bar{d}_1}, \ P = \frac{\bar{d}_1^2}{c\lambda\mu_0}\bar{P}, \ \dot{\gamma} = \frac{\bar{\dot{\gamma}}\bar{d}}{c} \tag{3}$$

and using the above non-dimensional quantities, the resulting governing equations become (Nadeem[18]),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\delta \mathrm{Re}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} - \delta^2 \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - \sigma^2 (u+1) \tag{5}$$

$$\delta^3 \mathrm{Re}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial x} - \delta \frac{\partial \tau_{yy}}{\partial y} \tag{6}$$

where

$$\sigma^2 = \frac{\in}{Da}, \quad Da = \frac{k}{a^2}, \ \in = porosity \quad and \quad k = permeability$$

$$\tau_{xx} = -2[1 + We\dot{\gamma}]\frac{\partial u}{\partial x},$$

$$\tau_{xy} = -[1 + We\dot{\gamma}]\left(\frac{\partial u}{\partial x} + \delta^2 \frac{\partial v}{\partial x}\right), \ \tau_{yy} = -2[1 + We\dot{\gamma}]\frac{\partial v}{\partial y},$$

$$\dot{\gamma} = \left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x}\right)^2 + 2\delta^2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}}.$$

Here δ, Re, We represent the wave, Reynolds and Weisseing numbers, respectively. Under the assumptions of long wavelength δ << 1 and low Reynolds number, neglecting the terms of order δ and higher, equations (5) and (6) take the form

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\left(1 + We\left(\frac{\partial u}{\partial y}\right)\right)\frac{\partial u}{\partial y}\right] - \sigma^2 (u+1) \tag{7}$$

$$\frac{\partial p}{\partial y} = 0 \tag{8}$$

The corresponding boundary conditions in wave frame of reference are given by

u= -1 on y=h_1(x)

u= -1 on y=h_2(x) (9)

Elimination of pressure from equations (7) & (8) yields

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left[\left(1 + We\left(\frac{\partial u}{\partial y}\right)\right)\frac{\partial u}{\partial y}\right] - \sigma^2 (u+1) \tag{10}$$

The volume flow rate q in a wave frame of reference is given by

$$q = \int_{h_2(x)}^{h_1(x)} u\,dy \tag{11}$$

The instantaneous flow Q(x, t) in a fixed frame is

$$Q(x,t) = \int_{h_2(x)}^{h_1(x)} (u+1)\,dy = \int_{h_2(x)}^{h_1(x)} u\,dy + \int_{h_2(x)}^{h_1(x)} 1\,dy = q + (h_1 - h_2) \tag{12}$$

The time average flux $\overline{Q}$ over one period T (= λ / c) of the peristaltic wave is

$$\overline{Q} = \frac{1}{T}\int_0^T Q\,dt \ = \frac{1}{T}\int_0^1 (q + h_1 - h_2)\,dt = \ q + 1 + d \tag{13}$$

III. Perturbation solution

Since, equation (10) is non-linear; its exact solution may not be possible. Therefore, we expand u, P and q as

$$u = u_0 + Weu_1 + 0(We^2), \ P = P_0 + WeP_1 + 0(We^2),$$

$$q = q_0 + Weq_1 + 0(We^2), \tag{14}$$

where $P = \frac{\partial p}{\partial x}$, Substituting above expressions in equation (10) and boundary conditions (9), we get the following system.

System of order We^0

$$\frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - \sigma^2 (u_0 + 1), \tag{15}$$

and the respective boundary conditions are

$$\mathrm{u}_0 = -1 \quad \text{for } \mathrm{y} = \mathrm{h}_1, \quad \mathrm{u}_0 = -1 \quad \text{for } \mathrm{y} = \mathrm{h}_2 \tag{16}$$

System of order We^1

$$\frac{dp_1}{dx}=\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial}{\partial y}\left(\frac{\partial u_0}{\partial y}\right)^2-\sigma^2 u_1 \qquad (17)$$

$$u_1=0, \quad \text{for } y=h_1,\ u_1=0, \quad \text{for } y=h_2 \qquad (18)$$

Solution for system of order We^0

Solution of Eq. (15) satisfying the boundary conditions (16) can be written as

$$u_0=\frac{p_0}{\sigma^2}\left[\left(\frac{1}{e^{\sigma h_1}}(1-ke^{-\sigma h_1})\right)e^{\sigma y}+ke^{-\sigma y}-1\right]-1$$

(19)

where $k=\left(\frac{\left(e^{\sigma h_2}-e^{\sigma h_1}\right)}{e^{\sigma(h_2-h_1)}-e^{\sigma(h_1-h_2)}}\right)$, and the volume flow rate q_0 is given by

$$q_0=\left(\frac{dp_0}{dx}\right)\left(\frac{1}{\sigma^3}\right)\left[e^{-\sigma h_1}\left(1-ke^{-\sigma h_1}\right)\left(e^{\sigma h_1}-e^{\sigma h_2}\right)-k\left(e^{-\sigma h_1}-e^{-\sigma h_2}\right)-\sigma(h_1-h_2)\right]-(h_1-h_2)$$

(20)

From Equation (20), we get

$$\frac{dp_0}{dx}=\frac{\sigma^3\left(q_0+(h_1-h_2)\right)}{e^{-\sigma h_1}\left(1-ke^{-\sigma h_1}\right)\left(e^{\sigma h_1}-e^{\sigma h_2}\right)-k\left(e^{-\sigma h_1}-e^{-\sigma h_2}\right)-\sigma\left(h_1-h_2\right)}$$

(21)

Solution for system of order We^1

Substituting the zeroth-order solution (19) into (17), the solution of the resulting problem satisfying the boundary conditions take the following form.

$$u_1=\left[\frac{(A+B+C)e^{-\sigma h_1}+D(E+F-G)e^{-\sigma h_1}}{D}\right]e^{\sigma y}-\left[\frac{A+B+C}{D}\right]e^{-\sigma y}-\frac{1}{\sigma^2}\left(\frac{dp_1}{dx}\right)$$
$$-Fe^{-2\sigma h_1}e^{2\sigma y}+\frac{2}{3\sigma^3}\left(\frac{dp_0}{dx}\right)^2k^2e^{-2\sigma y}$$

(22)

where A $=E\left(e^{\sigma h_1}-e^{\sigma h_2}\right)$, B $=$ F$\left(e^{-2\sigma h_1}e^{\sigma(2h_2-h_1)}-e^{\sigma h_2}\right)$,

C $=\frac{2}{3\sigma^3}\left(\frac{dp_0}{dx}\right)^2k^2\left(e^{\sigma(h_2-2h_1)}-e^{\sigma(h_1-2h_2)}\right)$

D $=e^{\sigma(h_2-h_1)}-e^{\sigma(h_1-h_2)}$, E $=\frac{1}{\sigma^2}\left(\frac{dp_1}{dx}\right)$,

F $=\frac{2}{3\sigma^3}\left(\frac{dp_0}{dx}\right)^2\left(1-ke^{-\sigma h_1}\right)^2$, G $=\frac{2}{3\sigma^3}\left(\frac{dp_0}{dx}\right)^2k^2e^{-2\sigma h_1}$

and the volume flow rate q_1 is given by

$$q_1=\frac{A}{D}e^{-2\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)+\frac{A}{D}e^{-2\sigma h_1}\left(\frac{e^{-\sigma h_1}-e^{-\sigma h_2}}{\sigma}\right)-E(h_1-h_2)$$
$$+Ee^{-\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)+H$$

(23)

where

$$H=\frac{(B+C)}{D}e^{-2\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)+(F-G)e^{-\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)+\frac{(B+C)}{D}\left(\frac{e^{-\sigma h_1}-e^{-\sigma h_2}}{\sigma}\right)$$
$$-\frac{F}{2\sigma e^{2\sigma h_1}}\left(e^{2\sigma h_1}-e^{2\sigma h_2}\right)-\frac{dp_0}{dx}\frac{k^2}{3\sigma}\left(e^{-2\sigma h_1}-e^{-2\sigma h_2}\right)$$

From equation (23), we get $\frac{dp_1}{dx}=\frac{\sigma^3\left(q_1-H\right)D}{I}$

(24)

where

$$I=\left(e^{\sigma h_1}-e^{\sigma h_2}\right)^2e^{-2\sigma h_1}+\left(e^{\sigma h_1}-e^{\sigma h_2}\right)\left(e^{-\sigma h_1}-e^{-\sigma h_2}\right)-\sigma D(h_1-h_2)+De^{-\sigma h_1}\left(e^{\sigma h_1}-e^{\sigma h_2}\right)$$

Substituting equations (21) and (24) in to equation (14) and using the relation (14), we get

$$\frac{dp}{dx}=\frac{\sigma^3}{J}\left[\left(q+(h_1-h_2)\right)-\left(\sigma^6We^2\left(q^2+h_1^2+h_2^2-2h_1h_2+2q(h_1-h_2)\frac{L}{J^2}\right)\right)\right] \qquad (25)$$

where $J=\frac{I}{D}$, Integrate above equation over one wavelength, we get (21)

$$\Delta P=\int_0^1\frac{dp}{dx}dx=\int_0^1\frac{\sigma^3}{J}\left[\left(q+(h_1-h_2)\right)-\left(\sigma^6We^2\left(q^2+h_1^2+h_2^2-2h_1h_2+2q(h_1-h_2)\frac{L}{J^2}\right)\right)\right]dx$$

(26)

where $L=A_1+A_2+A_3-A_4+A_5+A_6-A_7-A_8$

$$A_1=\frac{2}{3\sigma^3D}\left(1-ke^{-\sigma h_1}\right)^2\left(e^{-2\sigma h_1}e^{\sigma(2h_2-h_1)}-e^{\sigma h_2}\right)\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right),$$

$$A_2=\frac{2}{3\sigma^3D}k^2\left(e^{\sigma(h_2-2h_1)}-e^{\sigma(h_1-2h_2)}\right)\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)$$

$$A_3=\frac{2}{3\sigma^3}\left(1-ke^{-\sigma h_1}\right)^2e^{-\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right),$$

$$A_4=\frac{2}{3\sigma^3}k^2e^{-2\sigma h_1}\left(\frac{e^{\sigma h_1}-e^{\sigma h_2}}{\sigma}\right)$$

$$A_5=\frac{2}{3\sigma^3 D}\left(1-ke^{-\sigma h_1}\right)^2\left(e^{-2\sigma h_1}e^{\sigma(2h_2-h_1)}-e^{\sigma h_2}\right)$$
$$\left(\frac{e^{-\sigma h_1}-e^{-\sigma h_2}}{\sigma}\right)$$

$$A_6=\frac{2}{3\sigma^3 D}k^2\left(e^{\sigma(h_2-2h_1)}-e^{\sigma(h_1-2h_2)}\right)\left(\frac{e^{-\sigma h_1}-e^{-\sigma h_2}}{\sigma}\right)$$

$$A_7=\frac{2}{6\sigma^4}\left(1-ke^{-\sigma h_1}\right)^2 e^{-2\sigma h_1}\left(e^{2\sigma h_1}-e^{2\sigma h_2}\right),$$

$$A_8=\frac{k^2}{3\sigma^4}\left(e^{-2\sigma h_1}-e^{-2\sigma h_2}\right)$$

IV. Results and Discussions

From equation (26) we have calculated the pressure difference as a function of $\overline{Q}$ for different values of permeability parameter σ and different phase differences ϕ for a fixed a=0.8, b=0.5, d=2, We=0.03 and is shown in figures (2) and (3).We observe that the larger the parameter σ the greater the pressure rise against which the pump works. We observe that for a given ΔP, the flux $\overline{Q}$ increases with increasing σ .For free pumping there is no difference in flux $\overline{Q}$ for increase in σ.We also observed that the pressure rise increases with increasing phase difference ϕ.

The variation of pressure rise with time averaged flow rate is calculated from equation (26) for different values of We and different phase differences for a fixed a=0.8,b=0.5,d=2, σ=1.5 and is shown in figures (4) to (6). We observe that the larger the We, the smaller the pressure rise against which the pump works. We observe that for a given ΔP,the flux $\overline{Q}$ decreases with increasingWe. For a given flux $\overline{Q}$ the pressure rise decreases with increase We.

The variation of ΔP with time averaged flow rate is calculated from equation (26) for different values of the phase difference ϕ, for a fixed a=0.8, b=0.5,d=2, σ=0.5, We=0.03 and is shown in figure (7). We observe that the larger the phase difference ϕ, the greater the pressure rise against which the pump works. We observe that for a given ΔP, the flux $\overline{Q}$ increases with increasing ϕ. For a given flux $\overline{Q}$ the pressure rise increases with increasing ϕ.

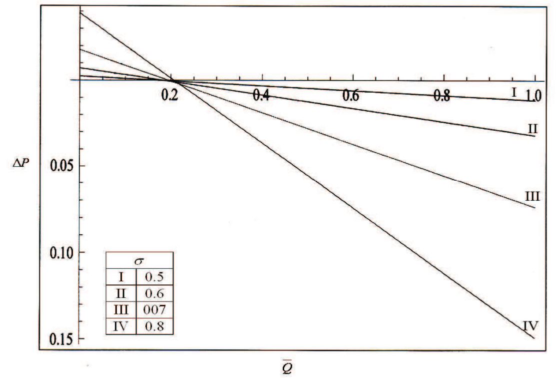

Fig.2. The variation of ΔP with $\overline{Q}$ for different values of σ for a fixed a = 0.8, b =0.5, ϕ =0, d =2; We = 0.03

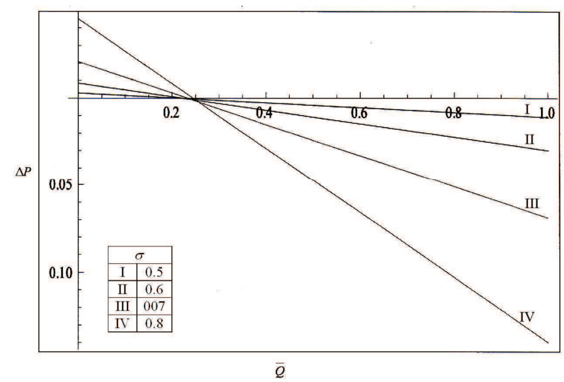

Fig.3. The variation of ΔP with $\overline{Q}$ for different values of σ for a fixed a = 0.8,b =0.5, ϕ = π/6, d =2; We = 0.0

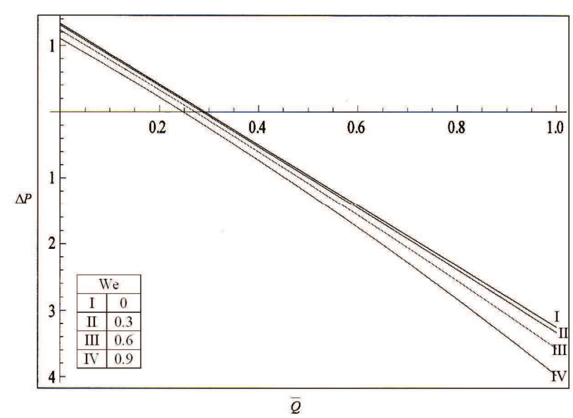

Fig.4. The variation of ΔP with $\overline{Q}$ for different values of We for a fixed a = 0.8,b =0.5, ϕ = 0, d =2; σ = 1.5.

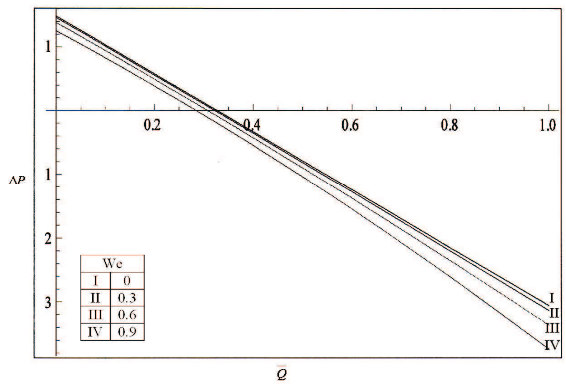

Fig.5. The variation of ΔP with $\overline{Q}$ for different values of We for a fixed a = 0.8, b =0.5, $\phi = \pi/6$, d =2, σ = 1.5.

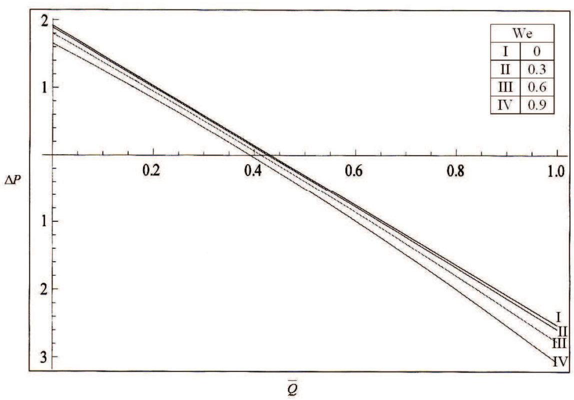

Fig.6. The variation of ΔP with $\overline{Q}$ for different values of We for a fixed a = 0.8, b =0.5, $\phi = \pi/3$, d =2, σ = 1.5.

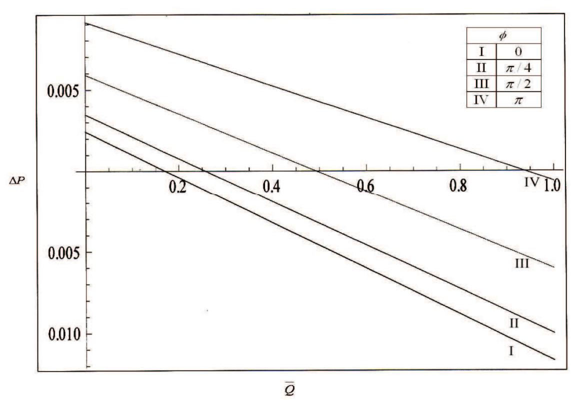

Fig.7. The variation of ΔP with $\overline{Q}$ for different values of ϕ for a fixed a = 0.8, b =0.5, We =0.03, d =2, σ = 0.5.

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